(1) Do problems 2.36 (viii)-(x), 2.37, 2.42, 2.44, 2.46, and 2.52 (i)-(iv) in the book.

(2) This problems will discuss the group $S_3 = S\{1,2,3\}$.

Let $S = \{1,2,3\}$. Let $\psi: S \to S$ be given by

$$\psi(1) = 2, \psi(2) = 3, \psi(3) = 1.$$ 

Let $\phi: S \to S$ be given by

$$\phi(1) = 2, \phi(2) = 1, \phi(3) = 3.$$ 

(a) Compute the following functions on $S$. (That is, tell what they do 1,2, and 3):

- $\psi^2 = \psi \circ \psi$,
- $\psi^3 = \psi \circ \psi \circ \psi$,
- $\phi^2$,
- $\phi^3$,
- $\psi \circ \phi$,
- $\phi \circ \psi$,
- $\psi^{-1}$,
- $\psi^{-1} \circ \phi$,
- $\psi^2 \circ \phi$.

Are any of these functions the same as each other?

(b) Note that the identity function $e$, $\psi$, $\phi$, $\psi^2$, $\psi \circ \phi$, and $\psi^2 \circ \phi$ are all distinct. Explain why this means that every element of $S_3$ must be equal to one of these.

(c) Make a group table for $S_3$ in which you label the rows and columns $e$, $\psi$, $\phi$, $\psi^2$, $\psi \circ \phi$ and $\psi^2 \circ \phi$ (Can you manage to do some (or even all!) of the products “algebraically” without having to go back to see what the functions do to 1, 2, and 3? Is this easier than doing it out by brute force as in last week’s assignment?)